

Programlama -1

“Symbolic Mathematics”
Dr. Cahit Karakuş, 2020

“Symbolic” toolbox allows you to:

- İfadeleri sembolik veri türleriyle sembolik biçimde girilir.
- Sembolik ifadeleri genişletir veya basitleştirir.
- Sembolik kökleri, limitleri, minimumları, maksimumları vb. bulur.
- Sembolik işlevleri ayırt eder ve bütünleştirir.
- Taylor fonksiyon serileri oluşturur (diğer araçların yanı sıra).
- Cebirsel ve diferansiyel denklemleri sembolik olarak çözer.
- Eşzamanlı denklemleri çözer (hatta bazı doğrusal olmayan).
- Değişken hassas aritmetik yapar.
- Sembolik fonksiyonların grafik gösterimlerini oluşturur.

Manipulating symbolic expressions

- `numden()`
- `expand()`
- `factor()`
- `collect()`
- `simplify()`
- `simple()`
- `poly2sym()`

numden()

- The `numden()` command is used to separate the numerator and denominator of a quotient.
- Bir bölümün payını ve paydasını ayırmak için `numden()` komutu kullanılır. Aşağıdaki ifadede `numden()` kullanın:

```
y=2*(x + 3)^2/(x^2 + 6*x + 9)
[numerator, denominator]=numden(y)
```

```
numerator= 2*(x + 3)^2
denominator= (x^2 + 6*x + 9)
```

expand()

- **expand()** is used to expand an expression by expanding the products of factors in an expression.

- Expand the numerator of y:

```
expand (numerator)
```

```
ans= 2*x^2 + 12*x + 18
```

factor()

- The `factor()` command is used to factor an expression into a product of terms.
- `factor()` komutu, bir ifadeyi bir terimler çarpımına ya da çarpanlarına ayırmak için kullanılır.

Example: Factor the expression for the denominator.

```
factor(denominator)
```

```
ans=
```

```
(x + 3)^2
```

simplify()

- The `simplify()` command uses the Maple simplification algorithm to simplify each part of an expression.
- Enter the expression:

```
b=sym('3*a - (a + 3)*(a - 3)^2');  
simplify(b)
```

```
ans= 12*a - a^3 + 3*a^2 - 27
```

simple()

- The `simple()` command executes many techniques to simplify an expression; its `ans` is the simplest expression.

```
simple(b)
```

```
ans = 3*a - (a + 3)*(a - 3)^2
```

Note: The `simple()` command displays the results of all simplification methods; only the final result, i.e., the answer (`ans`) is shown here.

poly2sym()

- The `poly2sym()` function uses an array of coefficients to create a polynomial:

```
a=[1,3,2];  
b=poly2sym(a)
```

```
b=  
x^2 + 3*x + 2
```

- The function `sym2poly()` is the inverse of `poly2sym()`.

solve()

- The `solve()` function sets an expression equal to zero, then solves the equation for its roots.

```
E1=x^2 - 9  
solve(E1)
```

```
ans=  
3  
-3
```

```
clear all  
close all  
a=[1 3 2]  
b=poly2sym(a)  
c=solve(b)
```

Hands on

```
solve('a*x^2 + b*x + c')
```

`ans =`

`1/2/a*(-b + (b^2 - 4*a*c)^(1/2))`

`1/2/a*(-b - (b^2 - 4*a*c)^(1/2))`

Note: The expression is in single quotes; if the symbolic variables in the expression have not been defined previously, then the single quotes are necessary.

Hands on

```
solve('a*x^2 + b*x + c', 'a')
```

```
ans =  
-(b*x + c)/x^2
```

Note that this solves the expression for "a".

Systems of equations

- You can use `solve()` to find the solution of a system of equations.

```
one=sym('3*x + 2*y - z = 10');  
two=sym('-x + 3*y + 2*z = 5');  
three=sym('x - y - z = -1');  
[x,y,z]=solve(one,two,three)
```

x=
-2

y=
5

z=
-6

```
clear all  
close all  
syms x y z
```

```
d1='3*x+2*y-z=10'
```

```
d2='-x+3*y+2*z=5'
```

```
d3='x-y-z=-1'
```

```
[x,y,z]=solve(d1,d2,d3)
```

Note that the `solve` function produces symbolic output. You can change the output to numerical values with the `double()` command.

Substitution with subs()

- The `subs()` command allows you to substitute a symbol with another symbol or assign a number to a variable.

```
syms a b c x y;  
quadratic = a*x^2 + b*x + c;  
yquadratic = subs(quadratic,x,y)
```

```
yquadratic =  
a*y^2 + b*y + c
```

Multiple substitutions

- You can use the `subs()` command to do multiple substitutions in one command. This is done by grouping the variables and their substitutes (other variables or numerical values) in braces.

```
subs(symbolic_function, {substituant}, {substitute})
```

```
subs(quadratic, {a,b,c,x}, {1,2,3,4})
```

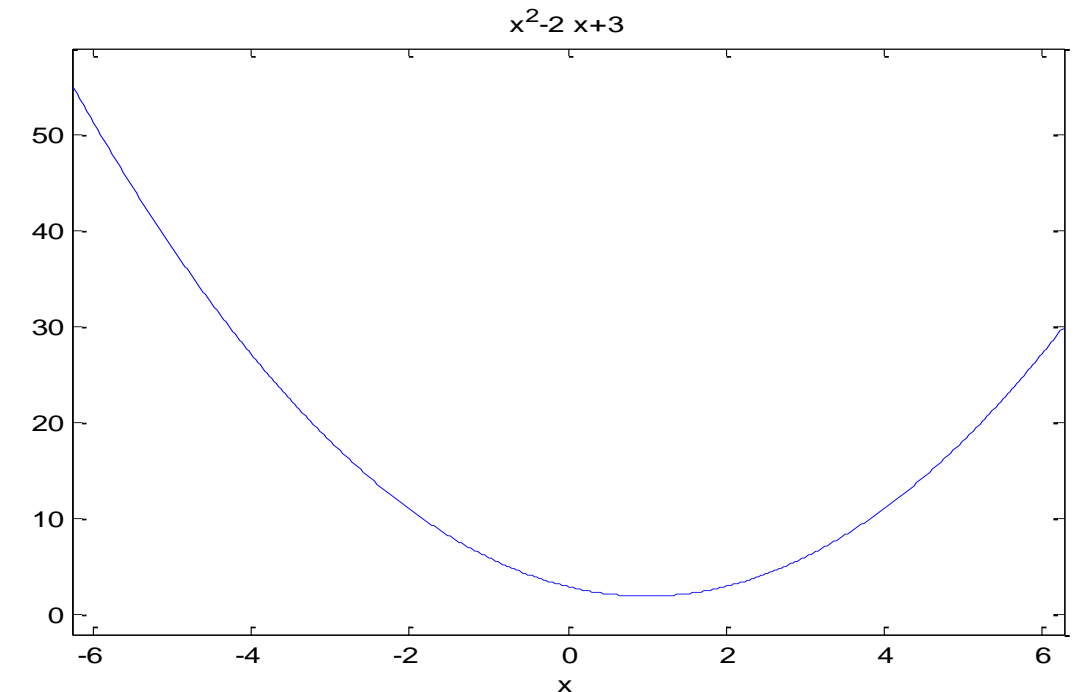
```
ans =
```

```
27
```

Plotting symbolical functions

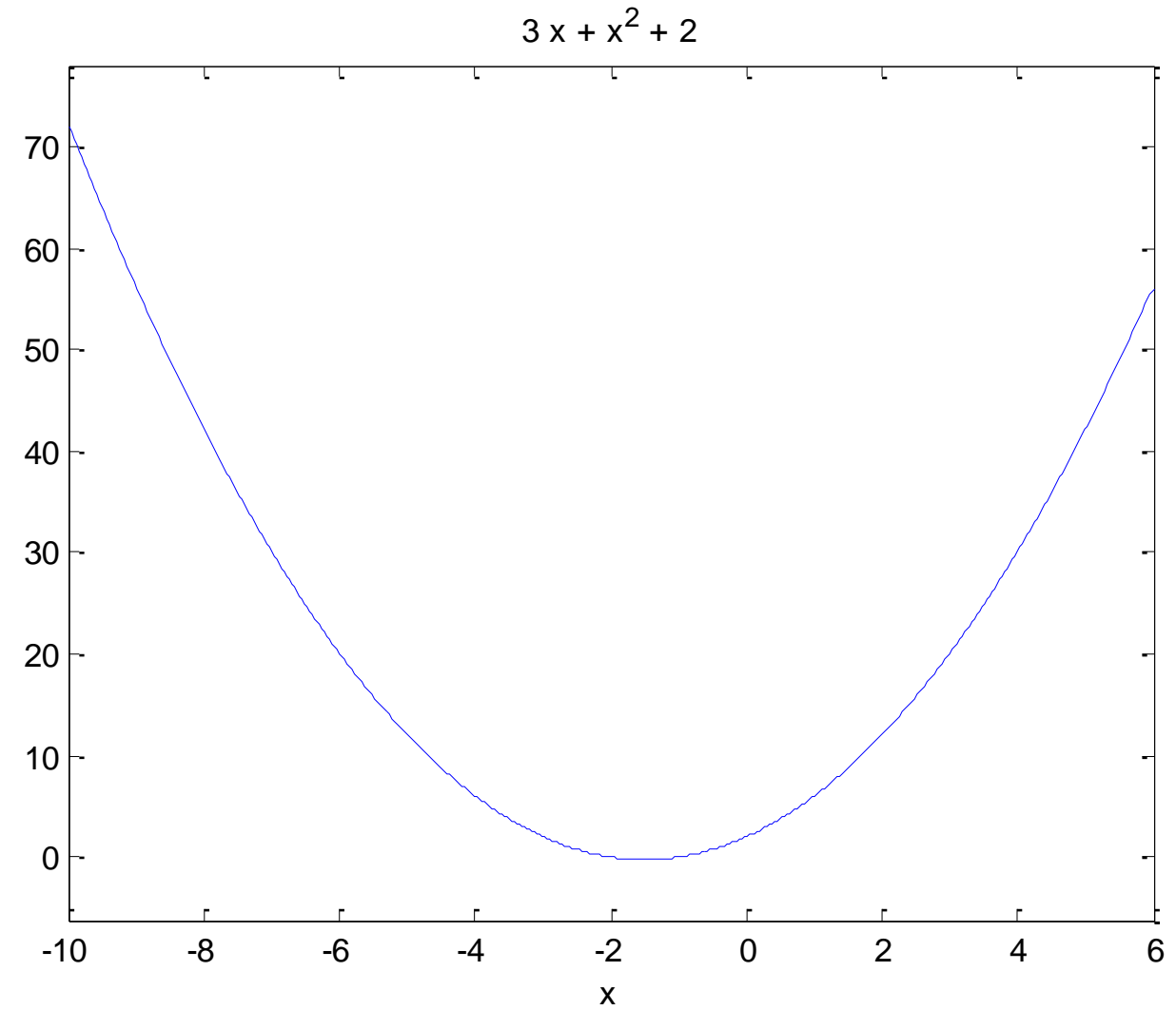
- Plotting symbolic functions in MATLAB is done with the `ezplot()` set of commands.
- The syntax of `ezplot()` when y is a function of x is:
`ezplot(y)`

```
y = sym('x^2 - 2*x + 3');  
ezplot(y)
```



EZPLOT

```
clear all  
close all  
a=[1 3 2]  
b=poly2sym(a)  
c=solve(b)  
ezplot(b, [-10, 6])
```



Differentiation

- MATLAB allows you to differentiate symbolic functions with the `diff()` command.
- The syntax of `diff()`, when `f` is a symbolic function of the variable `x` and `n` is the order of the derivative to be determined, is:

```
diff(f, 'x', n)
```

```
diff(y, 'x', 1)
```

```
ans =  
2*x-2
```

Try a second derivative. Your result should be 2.

Integration

- MATLAB allows you to integrate symbolic functions with the `int()` command. Either definite integrals, with assigned numeric or symbolic bounds, or indefinite integrals (note that when you compute indefinite integrals, the constant of integration does *not* appear explicitly).

```
int(y, 'x', a, b)
```

```
int(y, 'x')
```

```
ans =  
1/3*x^3-x^2+3*x
```

Try a definite integral for $1 < x < 2$. The result should be $7/3$.

Integration constant

- If you want to display the integration constant when applying the `int()` function, you can do the following:

```
syms x y a
```

```
y = 2*x;
```

```
int(y, 'a', 'x')
```

```
ans = x^2-a^2
```

Exercises

- Use the symbolic toolbox to solve the following system of equations:
$$x + 2y - z = 4$$
$$3x + 8y + 7z = 20$$
$$2x + 7y + 9z = 23$$
- The velocity of a car is $v = t^2 - 3t + 5$. Find the displacement for $1 < t < 5$ and the acceleration at $t = 1.5$. Plot the equations for distance, velocity, and acceleration on one graph.

Summary

- Creating Symbolic Expressions
 - `sym('x')`, `syms x`, expressions i.e. `e=sym('m*c^2')`
- Manipulation
 - `numden`, `expand`, `factor`, `collect`, `simplify`, `simple`, `poly2sym`
- Solutions
 - `solve`, `subs`
- Plotting
 - `ezplot`
- Differentiation
 - `diff(y, 'x', n)`
- Integration
 - `int(y, 'x', a, b)`